Automatic Microwave Q Measurement for Determination of Small Attenuations

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Abstract—The Q's of a waveguide transmission cavity with and without insertion of a length of waveguide give the insertion loss. Least-squares fitting of transmission versus frequency data have been used to determine Q and resonant frequency f_r with computer-controlled equipment in an iterative process. Preliminary values of Q and f_r are used to calculate an improved set of test frequencies, and so on. A rapid least-squares method is described which minimizes truncation errors.

Introduction

WAVEGUIDE RUNS frequently include many relatively short components. Lengths of flexible guide are often necessary. The cumulative attenuation of such components is often a significant system parameter. Assembly is performed in the field at significant expense. If an overall measurement suggests that a high-loss component has been included, the identification of the inferior component by substitution can be difficult and costly, especially if components were delivered in lots which might all be afflicted by the same defect.

For this reason, inspection testing of the loss of individual components is desirable. The entire quantity being measured, e.g., 0.1 dB, could be obscured by the uncertainty in the ordinary single-frequency method of using a commercial automatic network analyzer. Therefore, the waveguide component is made part of a transmission cavity and the Q of the cavity if determined. The cavity method gives the *dissipative* losses of the waveguide.

The effect of line loss on the Q of such a cavity has long been understood but has not been widely advocated as a basis for loss determination because of uncertainty about the additional dissipative loss in the irises [1]. In the method to be described, this problem is attacked simply by measuring the loss of the irises (also by the cavity method) and correcting approximately for these losses. A simple but key feature is that the iris-to-cavity contacts are not disturbed in the procedure; connections are made at waveguide flanges removed from the irises.

The method is not restricted to waveguide components, although they constitute an important application of their generally low values of loss. The general principles of the procedure will be described and followed by a discussion of algorithms for automatic Q measurement.

Manuscript received April 27, 1971; revised August 6, 1971.
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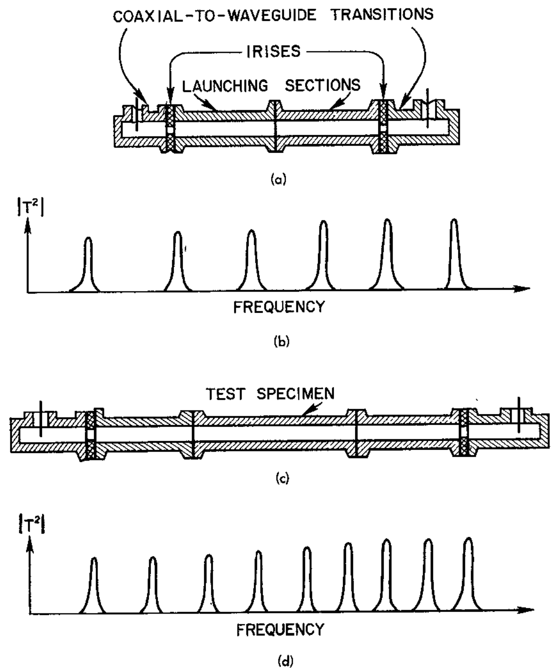

Fig. 1. (a) Initial cavity. (b) Initial cavity transmission. (c) Cavity with specimen. (d) Transmission of cavity with specimen.

Procedure

The microwave hardware configurations and corresponding transmission characteristics are indicated schematically in Fig. 1. A coaxial-to-waveguide input transition, an iris, a "launching" length of waveguide, a "receiving" length of waveguide, another iris, and a waveguide-to-coaxial output transition are shown in Fig. 1(a). Initially, the launching and receiving waveguides are connected together to form a cavity. The transmission characteristics are shown in Fig. 1(b). The resonant frequency, peak value of transmission, and Q of one or more resonances are determined as described below.

The piece to be tested is inserted as shown in Fig. 1(c) to give the transmission characteristic shown in Fig. 1(d). The spacing between transmission peaks is narrowed by the increased electrical length. Resonant frequency, Q, and peak transmission are again determined in the desired frequency range.

An exact analysis of the measurement situation would require the scattering parameters of each of the irises. However, it is hardly practical to hope for better than 10-percent accuracy in the measurement of the dissipation, and that only if the irises have small transmission. The following approximate analysis permits a simple understanding of the method.

Analysis

For each cavity, the voltage decrement δ is related to Q by

$$\delta = \frac{\omega}{2Q} \tag{1}$$

so that the voltage is reduced in one cycle by the factor

$$e^{-\delta/f} = \exp\left(-\frac{\omega}{2Q}\cdot\frac{2\pi}{\omega}\right) = e^{-\pi/Q} \tag{2}$$

or, in decibels,

$$20 \log_{10} e^{-\pi/Q} = -\frac{20\pi}{Q}\log_{10} e = -\frac{27.3}{Q}\ \frac{\text{dB}}{\text{cycle}} \tag{3}$$

which can be interpreted as $27.3/Q$ dB loss per guide wavelength.

Let L_1 and L_2 be the physical lengths of the first and second cavities and λ_{g1} and λ_{g2} be the guide wavelengths at the resonant frequencies. Then the apparent attenuations are $27.3\ L_1/(Q_1\lambda_{g1})$ and $27.3\ L_2/(Q_2\lambda_{g2})$. Both values include the loss contributions of the irises. The loss of the short cavity is subtracted from the loss of the long cavity to obtain the loss of the inserted length of waveguide:

$$\text{insertion loss (dB)} = 27.3\left(\frac{L_2}{Q_2\lambda_{g2}} - \frac{L_1}{Q_1\lambda_{g1}}\right). \tag{4}$$

Strictly speaking, the two cavities should have the same resonant frequency or interpolation should be used. However, a 5-percent difference in frequency is rarely significant in comparison to other sources of inaccuracy.

Since the irises are near-short-circuits, the end-current distributions for each cavity will be similar, which is another requirement for the validity of the subtraction.

This determination gives dissipative loss only. Waveguide connections made with normal care usually have negligible reflection losses. When there is any doubt about this matter, nonresonant reflection measurements can be made by standard methods.

Algorithm for Q Measurement

The phase-locked version of the Hewlett-Packard 8542A network analyzer system can be set on a digitally specified frequency with more than adequate accuracy and resolution for determining the highest Q values likely to be encountered in waveguide-attenuation evaluation. However, the time required for setting frequency and taking a measurement is about 250 ms.

A simple search for the frequencies of maximum transmission and 3-dB loss, mimicking typical manual techniques, would be prohibitively time-consuming. At the same time, the computer capability of the system should permit basing the results on more than three measurements so that accuracy can be improved over manual methods.

One technique that is simple to program is based on the principle that group delay peaks at the resonant frequency and the Q is proportional to the peak group delay. In its simplest form, however, this method does not make use of more than two or three measurement results. Also, the Q value is distorted by line lengths, unless corrections are applied.

A larger number of measurements could be involved in a procedure of fitting a curve to the complex transmission coefficient. Again, however, line length adds an extravaneous parameter to the problem.

The method now used in both the search and the final Q determination is to fit a theoretical resonance curve to the transmission *magnitude* measurements. Since phase data are disregarded, line lengths are unimportant.

Instead of the exact formulation for the multiple resonances of the waveguide cavity, the theoretical curve for a simple resonance is adequate:

$$|T^2| = \frac{4R_gR_L}{(R_s + R_L + R_g)^2 + \left(2\pi fL - \dfrac{1}{2\pi fC}\right)^2} \tag{5}$$

where

$$R_L = R_g \ll R_s \tag{6}$$

is the favorable condition for this measurement (lightly loaded symmetrical cavity).

The loaded $Q(Q_l)$ obtained from this measurement can be corrected to the unloaded $Q(Q_u)$ by the relation

$$Q_u = Q_l/(1 - |T_{\max}|). \tag{7}$$

If the peak transmission $|T_{\max}|$ corresponds to a 30-dB loss, the difference between Q_u and Q_l is 3 percent.

To obtain true transmission coefficient T, the raw complex transmission measurement R would have to be corrected by dividing by the raw calibration measurement R_t obtained on a through connection:

$$T = R/R_t. \tag{8}$$

Since the exact measurement frequencies are not known beforehand, it is practical to work with the raw-measurement values on the assumption that $|R_t|$ does not vary appreciably over the bandwidth of the resonance. In fact, under this assumption one can determine f_r and

Q without any measurement of R_t. The resonance curve (6) can be cast in the form of a polynomial in f^2 by treating $f^2/|R|^2$ as the dependent variable:

$$\frac{f^2}{|R|^2} = \frac{1}{16\pi^2 R_g^2 C^2 |R_t|^2} + \frac{f^2}{|R_t|^2}\left[\left(1 + \frac{R_s}{2R_g}\right)^2 - \frac{L}{2R_g^2 C}\right] + \frac{4\pi^2 f^2 L^2}{R_g^2 |R_t|^2} \quad (9)$$

where it has been assumed that $R_L = R_g$. By the substitution $x = f^2$ and $y = 1/|R|^2$, this equation can be written

$$xy = a_0 + a_1 x + a_2 x^2. \quad (10)$$

A least-squares determination of the coefficients a_0, a_1, and a_2 is adequate for the measurement procedure and simple to implement with one precaution.

Roundoff error is a major pitfall in least-squares calculations when the variation in the dependent variable is small in comparison to its absolute value. For this reason, one must transform to a new independent variable. Also, for computational simplicity, this variable can be an integer K. Thus let

$$x = G + SK, \qquad -N \le K \le N \quad (11)$$

where $\sqrt{G}$ is the estimated resonant frequency (initial guess or previous result) and S is a step size (in squared frequency) appropriate to the bandwidth of the resonance. Then (10) becomes

$$\begin{aligned} G_y + SK_y &= a_0 + a_1 G + a_2 G^2 + (a_1 + 2a_2 G)SK + a_2 S^2 K^2 \\ &= C_0 + C_1 K + C_2 K^2. \end{aligned} \quad (12)$$

The coefficients C_0, C_1, and C_2 can be found by solving the moment equations

$$\begin{aligned} G\sum y_k + S\sum Ky_k &= C_0 \sum 1 + 0 + C_2 \sum K^2 \\ G\sum Ky_k + S\sum K^2 y_k &= 0 + C_1 \sum K^2 + 0 \\ G\sum K^2 y_k + S\sum K^3 y_k &= C_0 \sum K^2 + 0 + C_2 \sum K^4 \end{aligned} \quad (13)$$

where the summation extends from $K = -N$ to $+N$. The summations on the right-hand side of (13) can be written in closed form as polynomials in N. The linear equations (13) can be solved for arbitrary values of N. However, it is much simpler to choose a particular value of N (8 is used in the sample program). Also, the solution for a_0, a_1, and a_2 from C_0, C_1, and C_2 is obvious from (12). For the chosen value of N, all of these calculations can be carried out in five basic statements with appropriate rational constants.

The resonant frequency f_r and Q are calculated from

$$\begin{aligned} |R_t|^2 a_0 &= \frac{1}{16\pi^2 R_g^2 C^2} \\ |R_t|^2 a_1 &= \left(1 + \frac{R_s}{2R_g}\right)^2 - \frac{L}{2R_g^2 C} \\ |R_t|^2 a_2 &= \frac{4\pi^2 L^2}{R_g^2} \end{aligned} \quad (14)$$

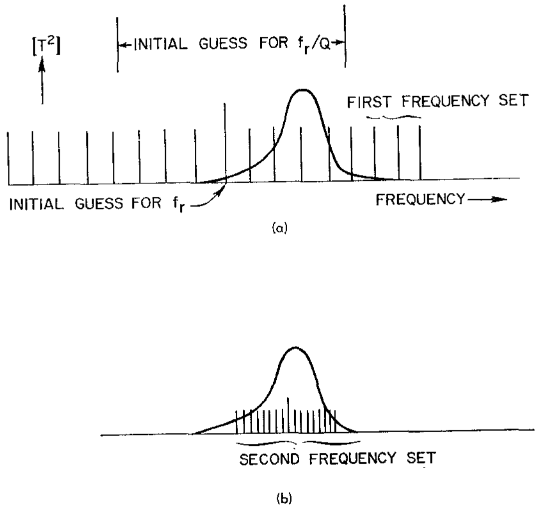

Fig. 2. (a) First deployment of measurement frequencies based on deliberately low estimate of Q. (b) Second set of measurement frequencies often results in a computed f_r and Q substantially equal to final result.

so that

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \sqrt[4]{\frac{4a_0}{a_2}} \quad (15)$$

$$Q_l = (2 + a_1/\sqrt{a_0 a_2})^{-1/2}. \quad (16)$$

For accuracy, frequencies reasonably close to resonance should be used; otherwise, undue weight will be given to the least accurate readings since the reciprocal of transmission appears in (9). The sample program settles down to a frequency range corresponding to twice the bandwidth.

If correction for loading is desired in accordance with (7), one can determine the peak transmission $|T_{max}|$ from (8) and

$$1/|R_{max}|^2 = a_1 + 2\sqrt{a_0 a_1}. \quad (17)$$

In the program the correction for loading is based on the assumption that the through reading magnitude will be a slowly varying function of frequency. Two points and linear interpolation give $|T_{max}|$ with adequate accuracy for the loading correction if the cavity is reasonably well decoupled.

In operation a first estimate of f_r and a decidedly low estimate of Q are given. The resulting f_r and Q are used in selecting the frequencies for the next set of measurements, as shown in Fig. 2, and so on until stable values are found.

Results

This program has been applied both to waveguide cavities and to coaxial cavities containing mechanical and semiconductor variable capacitors. The Q's range from 250 to 4000. Two to six iterations were required for convergence (depending upon the initial guess), after which further iterations generally varied stably within a 3-percent range.

A 2-ft length of X-band waveguide (WR-90) was measured by this method and a dissipative loss of 0.12 ± 0.02 dB was found at 8700 MHz. If the correction for the short cavity had been neglected, the apparent loss would have been 0.18 dB. The irises were made from 0.020-in copper by punching 3/16-in diameter holes.

Conclusions

Since the losses of both irises and the launching and receiving sections are all lumped together, the correction for these losses is approximate. However, the waveguide-attenuation measurement is practical only when these losses are small enough so that the approximation introduces negligible error. At the same time, these losses are often too large to be neglected entirely.

The curve-fitting algorithm that has been described is capable of determining the Q of microwave circuits with as much accuracy and reproducibility as can be hoped for in dealing with practical circuits. The reproducibility of connections is generally the limiting factor in circuits of the type studies. (Special cavities could be designed as Q standards for more critical evaluation of reproducibility and for intercomparison of different instruments and methods.)

The algorithm has a modest ability to find and converge upon a resonance, if given a good initial guess, which is usually a simple matter in production testing of similar parts.

However, the search capability could be enhanced by using the measured transmission magnitude as a weighting factor. This technique would increase the computation required but would make better use of the measurements.

Acknowledgment

The author wishes to thank Computer Metrics, Inc., Rochelle Park, N.J., and Palo Alto, Calif., for making the automatic network analyzer time available.

References

[1] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits.* New York: McGraw-Hill, 1947.

Impedance Measurements of Microwave Lumped Elements from 1 to 12 GHz

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Abstract—**The impedance measurement of small, microwave lumped elements of the order of 1 mm has been extended up to 12 GHz by a technique in which the frequency and Q of a resonant transmission line are perturbed by the connection of a lumped element. With the use of low-loss resonant coaxial lines, the technique has been applied to the measurement of lumped-element capacitors ranging from 0.4 to 3.6 pF and inductors ranging from 1.1 to 4.3 nH. Conductor Q values for capacitors as high as 1700 at 1.4 GHz and 100 at 12 GHz have been measured and estimates of dielectric Q values for capacitors of over 5000 have been obtained. Single-turn 1.1-nH inductor Q's of 40 at 1 GHz and 90 at 7 GHz have also been measured. The capacitors and single-turn inductors are found to have constant C and L values up to 12 GHz.**

Manuscript received March 4, 1971; revised September 9, 1971. The work in this paper was supported by the U.S. Army Electronics Command, Fort Monmouth, N. J., under Contract DAAB07-68-0296, and by RCA Laboratories, Princeton, N. J.

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I. Introduction

At MICROWAVE frequencies the increasing use of solid-state active devices and the trend toward smaller electronic packages has stimulated an interest in replacing distributed circuits with smaller sized lumped-element circuits. In the past the use of lumped elements has been limited in frequency to below 1 GHz by problems of fabrication and relatively high losses. However, because of recent advancements in the technology of thin-film fabrication [1], [2], low-loss lumped-element circuits capable of competing with distributed circuits have been made in the S- and C-band ranges [1]–[6]. Unfortunately, the losses of lumped elements operating in the gigahertz range have been difficult to measure. The smaller sizes and lower